③

Exercise 2.2.10

a) Show that $|x+y| \leq |x| + |y|$ for all $x, y \in \mathbb{R}$

$$(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})[x>0, y>0 \Rightarrow |x+y| = |x| + |y|]$$

$$(\forall x \in \mathbb{R})(\forall y \in \mathbb{R}[x>0, y<0 \Rightarrow |x+y| < |x| + |y|]$$

b) Show that $||x|-|y|| \leq |x-y|$ for all $x, y \in \mathbb{R}$

$$|x| = |(x-y)+y|$$

$$|(x-y)+y| \leq |x-y| + |y| \quad \text{(triangle inequality)}$$

$$|x| \leq |x-y| + |y|$$

$$|x| - |y| \leq |x-y|$$

~~since~~

$$\cancel{|y| - |x| \leq |x-y)}$$

~~we use symmetry to conclude that~~

$$\cancel{||x| - |y|| \leq |x-y|}$$

(4) Then $|y| = |(y-x)+x| \leqslant |y-x| + |x|$

So $|y| - |x| \leqslant |x-y|$

$\therefore \ |x| - |y| \geqslant -|x-y|$

So we have two inequalities:

$$|x| - |y| \geqslant -|x-y|$$
$$|x| - |y| \leqslant |x-y|$$

$$\therefore \ \big||x| - |y|\big| \leqslant |x-y|$$